**5. MAXIMA AND MINIMA**

**Critical Point:** Point at slop is not defined and point at which f’(x) = 0. Hence, f’(x) is 0 or undefined.

**Extreme Value Theorem:**

For a continuous function f(x) on [a, b] there exists Maxima and Minima.

* The maxima and minima values occur at either at the end point or at the critical point (f’(x) = 0).

For a continuous function f(x) on (a, b), maxima or minima may or may not exist.

* If maxima or minima exists, then will occur only at critical point.

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| **Concave Up/Down:**  Cone Cave  **Absolute/Local Maxima/Minima:**  **Local Maxima/Minima:** x at f’(x) = 0  **Point of Inflection:**  Point on the graph of a function at which concavity changes from up to down or vice versa.  1. f’’(x) = 0  2. f’’(x) should switch it’s sign about the inflection point.  No maxima or No minima at point of inflection. | Calculus - Maxima and Minima (solutions, solutions, videos) |

**Maxima And Minima For Function Of Two Variable:**

Let be the function of two variable for which maxima and minima is to be determined.

1. Find p, q, r, s, t.
2. Obtain stationary points by evaluating p and q to zero.
3. Find r, s, t at stationary points.

Concluding Remarks:

1. If , and , f (x, y) has Minimum at that stationary points.
2. If , and , f (x, y) has Maximum at that stationary points.
3. If , there is no extremum at that stationary points and such that point is called saddle points.
4. If test is un-conclusive. Need to go with higher order derivative.

All critical points are not stationary points but all stationary points are critical points.

**Lagrange Multiplier (Constrained Optimisation):**

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| --- | --- | --- |
| Type of Constrain | | |
| Unconstrained Optimization | Inequality Constrained Optimization | Equality Constrained Optimization |

When we want to Maximise (or Minimise)

Subjected to constrain that

Follow these steps,

1. Introduce a new variable λ and defined a new function as follows;

This function is called Lagrangian, and new variable is referred as Lagrange multiplier.

1. Set gradient of equal to zero vector.

In other words, find the stationary points of .

1. Consider each solution, substitute it in f.

Whichever one gives the greatest (or smallest) value is the maximum (or minimum) point you are seeking.

**Solution of function:** ax**2** + bx + c